Multi-Objective Genetic Algorithm for Heston Model Calibration: A Framework Integrating Pricing and Implied Volatility Criteria

Yan Fang[1(✉)] and Yiwei Hua [1] and Wenwen Pan[2] and Julius Wu[3]

[1] School of Statistics and Data Science, Shanghai University of International Business and Economics, Shanghai, China, 201620
[2] Shanghai Federation of Social Science Associations, Shanghai, China, 200020
[3] University of California, Davis, CA, United States, 95616
yiffanyfang163.com

Abstract. Accurate calibration of the Heston model is essential for reliable option pricing, yet it remains a challenging task due to the nonlinear structure of the model and the presence of multiple conflicting objectives. Most existing approaches rely on single-objective formulations that focus solely on price fitting, which may lead to suboptimal overall performance. To address this issue, this paper proposes a multi-objective genetic algorithm framework for Heston model calibration, in which option price errors and implied volatility errors are jointly optimized. The resulting bi-objective optimization problem is solved using two representative evolutionary algorithms, namely NSGA-II and NRGA. This formulation enables an explicit characterization of the trade-off between pricing accuracy and volatility fitting. Extensive experiments are conducted using real option data from SSE 50ETF and S&P 500 markets, as well as Monte Carlo simulations. The results show that both NSGA-II and NRGA produce consistent and comparable parameter estimates. Compared with conventional single-objective methods, the proposed framework leads to improved pricing accuracy and more stable calibration results. Simulation studies further confirm the robustness of the proposed approach under different noise conditions.

Keywords: Heston Model, Multi-objective Optimization, Genetic Algorithm, NSGA-II, NRGA, Option Pricing, Implied Volatility.

1 Introduction

Options play a key role in modern financial markets, particularly in risk management and asset allocation. The accuracy of option pricing is therefore a fundamental issue in both academic research and practical applications. The Black–Scholes (BS) model [1] provides a classical framework; however, its assumption of constant volatility fails to capture empirical features such as volatility smiles and skews. To address this limitation, stochastic volatility models have been developed, among which the Heston model [2] is one of the most widely adopted. In the Heston framework, volatility is modeled as a mean-reverting stochastic process, thereby enabling the model to capture important stylized facts of asset returns while maintaining analytical tractability.

Despite its theoretical appeal, the practical performance of the Heston model depends critically on the accurate calibration of its parameters. This calibration problem is well known to be challenging due to the nonlinearity of the pricing function and the presence of multiple local optima [2-3]. To address these challenges, a range of calibration methods has been developed in literature. These approaches can broadly be divided into two categories: gradient-based methods (e.g., [1], [4-6]) and metaheuristic optimization techniques (e.g., [7-10]). Among these methods, gradient-based methods may suffer from slow convergence and sensitivity to local optima, particularly in high-dimensional or non-convex settings. By contrast, metaheuristic approaches, such as the genetic algorithm (GA) introduced by [11], provide more flexible global search capabilities and are less dependent on initial values, thereby making them suitable for complex optimization landscapes. A growing body of evidence suggests that GA-based calibration can improve both estimation accuracy and computational efficiency in the Heston framework (e.g., [8], [12-13]).

Nevertheless, most existing studies formulate the calibration problem as a single-objective optimization task, typically focusing on minimizing pricing errors alone [14-16]. While such formulations can achieve satisfactory fits in terms of option prices, they may overlook relevant information contained in other performance measures [17]. These limitations motivate the need for multi-objective formulations, as many decision problems involve multiple competing criteria that cannot be adequately represented within a single-objective framework [18-20].

This issue is particularly pronounced in the context of option pricing. Minimizing discrepancies between model-implied and observed prices does not necessarily ensure an accurate representation of the implied volatility surface. This observation suggests incorporating implied volatility as a complementary calibration target. As emphasized by [21], implied volatility plays a significant role in option valuation. Similarly, [22] proposes calibration approaches that explicitly account for the implied volatility surface. Taken together, these findings suggest a multi-objective formulation of the calibration problem that jointly incorporates both price and implied volatility information. Multi-objective genetic algorithms provide an effective computational framework for solving such problems. Since the seminal work of [23], extensive research has advanced their development, including [24-26].

In this study, we formulate the calibration of the Heston model as a bi-objective optimization problem, in which pricing errors and implied volatility errors are treated as separate objectives, allowing the trade-offs between these two dimensions to be explicitly characterized. To solve this problem, we employ evolutionary algorithms, specifically the Non-dominated Sorting Genetic Algorithm II (NSGA-II) in [27] and the Non-dominated Ranked Genetic Algorithm (NRGA) in [28], both of which are based on Pareto dominance and ranking principles. While these algorithms have been widely applied in general optimization settings, their use in option pricing model calibration remains relatively limited.

The proposed framework is applied to option data from the SSE 50ETF and S&P 500 markets to assess its empirical performance. The results indicate that both NSGA-II and NRGA produce broadly consistent parameter estimates. Compared with conventional single-objective approaches, the multi-objective framework improves pricing accuracy. A Monte Carlo simulation study further supports the robustness of the proposed multi-objective framework.

The main contributions of this study are threefold. First, we develop a multi-objective calibration framework for the Heston model that reformulates parameter estimation as a multi-objective optimization problem. Second, we jointly incorporate option price errors and implied volatility errors as calibration objectives, thereby explicitly capturing the trade-off between price fitting and volatility fitting within a unified framework and overcoming the limitations of conventional single-objective approaches. Third, we implement the framework using NSGA-II and NRGA and demonstrate its effectiveness through empirical analysis based on option data from both developed and emerging markets, as well as simulation experiments. The results indicate that the proposed framework delivers stable, consistent, and robust pricing performance, with improvements relative to standard single-objective calibration methods.

The remainder of this paper is organized as follows. Section 2 presents the problem formulation of the Heston model. Section 3 describes the proposed multi-objective genetic algorithm framework. Section 4 reports experimental results based on real market data and simulation studies. Section 5 concludes the paper.

2 Problem Formulation

This section introduces the Heston model and formulates its multi-objective extension. The notation used throughout the paper is defined as follows. Let κ denote the speed of mean reversion, θ the long-term variance, σ the volatility of volatility, ρ the correlation between two Brownian motions, V_0 the initial value of instantaneous variance, and λ the market price of volatility risk.

2.1 Heston Option Pricing Model

The Heston model [2] describes the dynamics of the underlying stock price S_t through the stochastic diffusion process:

$$dS_t = \mu S_t dt + \sqrt{V_t} S_t dW_t^1, \tag{1}$$

where μ is the drift rate, V_t denotes the instantaneous variance, and W_t^1 is a standard Brownian motion. The variance process V_t follows a Cox-Ingersoll-Ross (CIR) process [29]:

$$\mathrm{d}V_t = \kappa(\theta - V_t)dt + \sigma\sqrt{V_t} dW_t^2, \tag{2}$$

where $\kappa > 0$, $\theta > 0$, and $\sigma > 0$, and W_t^2 is another standard Brownian motion satisfying $dW_t^1 dW_t^2 = \rho dt$.

Under the standard no-arbitrage principle [1], the price of a derivative contingent on S and V satisfies the partial differential equation (PDE):

$$\frac{1}{2}VS^2\frac{\partial^2 U}{\partial S^2} + \rho\sigma VS\frac{\partial^2 U}{\partial S \partial V} + \frac{1}{2}\sigma^2 V\frac{\partial^2 U}{\partial V^2} + rS\frac{\partial U}{\partial S} + \left(\kappa[\theta - V] - \lambda(S,V,t)\right)\frac{\partial U}{\partial V} - rU + \frac{\partial U}{\partial t} = 0, \tag{3}$$

where r is the risk-free interest rate, and $\lambda(S, V, t)$ is the market price of volatility risk. For a European call option written on the underlying asset S at time t, the option price $C(S, V, t)$ satisfies the PDE given in Equation (3).

Given the unobservable nature of the market price of volatility risk $\lambda(S, V, t)$, the Heston model posits a proportional relationship to volatility, such that $\lambda(S, V, t) = \lambda V$, where λ is a constant scaling factor. Under this assumption and in the absence of dividends, the European call option price $C(S, V, t)$ can be expressed by a closed-form expression, i.e.,

$$C(S, V, t) = SP_1 - Ke^{-r(T-t)}P_2, \tag{4}$$

where K is the strike price, T is the maturity date, P_1 and P_2 are the adjusted risk-neutral probability, and the standard risk-neutral probability, respectively. The corresponding European put option price follows directly from the put-call parity:

$$P(\mathrm{S}, \mathrm{V}, \mathrm{t}) = C(\mathrm{S}, \mathrm{V}, \mathrm{t}) + Ke^{-r(T-t)} - S. \tag{5}$$

The probabilities P_1 and P_2 are obtained via the characteristic function approach combined with an inverse Fourier transform [30]:

$$P_j = \frac{1}{2} + \frac{1}{\pi}\int_0^{+\infty} Re\left[\frac{e^{-iu\ln(K)} f_j(S, V, T, u)}{iu}\right] du, \quad j = 1,2, \tag{6}$$

where i is the imaginary unit, $Re[\cdot]$ extracts the real part, and f_j is the characteristic function of $x = \ln(S)$ under the respective measure, given by:

$$f_j(S, V, T, u) = e^{-iu\mathrm{x} + C_j(\tau, u) + D_j(\tau, u)V}, \tag{7}$$

where $\tau = T - t$, and

$$C_j(\tau, u) = rui\tau + \frac{\kappa\theta}{\sigma^2}\left(b_j - \rho\sigma ui + d_j\right) - 2\ln\left(\frac{1 - g_j e^{d_j\tau}}{1 - g_j}\right),$$

$$D_j(\tau, u) = \frac{b_j - \rho\sigma ui + d_j}{\sigma^2}\left(\frac{1 - e^{d_j\tau}}{1 - g_j e^{d_j\tau}}\right),$$

With $g_j = \frac{b_j - \rho\sigma ui + d_j}{b_j - \rho\sigma ui - d_j}$, $d_j = \sqrt{\left(\rho\sigma ui - b_j\right)^2 - \sigma^2\left(2a_j ui - u^2\right)}$, $a_1 = \frac{1}{2}$, $a_2 = -\frac{1}{2}$, $b_1 = \kappa + \lambda - \rho\sigma$, and $b_2 = \kappa + \lambda$.

The Heston model involves six parameters $\Omega = \{\kappa, \theta, \sigma, \rho, V_0, \lambda\}$. Setting $\lambda = 0$ under the risk-neutral measure reduces the parameter vector to $\{\kappa, \theta, \sigma, \rho, v(0)\}$. Since the put-call parity ensures that European call and put options convey equivalent pricing information, this study focuses on European call options and conducts all calibration and pricing analyses under the five-parameter specification.

2.2 Multi-Objective Calibration Formulation

Consider a decision-maker who seeks to optimize K non-commensurable objective functions in the absence of an explicit preference structure among them. Without loss of generality, all objectives are formulated as minimization problems since any maximization objective can be converted into an equivalent minimization form by multiplying it by -1. According to [31], the multi-objective optimization problem is stated as:

$$\min_{\mathbf{x}}\big(f_1(\mathbf{x}), f_2(\mathbf{x}), \cdots, f_K(\mathbf{x})\big), \quad (8)$$
$$\text{subjec to } \mathbf{x} \in S,$$

where $\mathbf{x}$ denotes the decision vector, $f_i(\mathbf{x})$ represents the ith objective function for $i = 1, \cdots, K$, and S is the feasible search space.

Within this general framework, the calibration of the Heston model can be viewed as a special case involving multiple competing criteria. Heston calibration requires balancing several aspects of model performance, including pricing accuracy, implied volatility consistency, and related market characteristics. These criteria are typically non-commensurable and cannot be adequately summarized by a single performance measure, which motivates a multi-objective formulation of the calibration problem.

In empirical option pricing, pricing accuracy [14] and implied volatility fitting accuracy [22] are widely regarded as the two most informative and economically relevant evaluation dimensions. Accordingly, this study concentrates on a bi-objective formulation of the Heston model calibration problem ($K = 2$), in which the objective functions correspond to pricing errors and implied volatility errors, respectively.

(1) Objective 1: Minimization of the weighted squared pricing error

Following [14], the first objective targets the minimization of the discrepancy between model-implied option prices and observed market prices, thereby offering a direct measure of pricing accuracy. Let N denote the sample size, and let $C_i^P(\Omega)$ and C_i^M represent the model-implied and market-observed prices of the i-th option, respectively. The objective function is defined as

$$f_1(\Omega) = \sum_{i=1}^{N} \omega_i (C_i^P(\Omega) - C_i^M)^2, \quad (9)$$

where ω_i denote weight assigned to the i-th option.

(2) Objective 2: Minimization of the weighted squared implied volatility error

Implied volatility plays a crucial role in option pricing and risk management [32]. In addition to pricing accuracy, it is essential to consider how well a model captures the implied volatility surface, as this provides further insights into market dynamics. This study thus integrates implied volatility as a second calibration objective to complement price-based fitting. The corresponding objective function is defined as:

$$f_2(\Omega) = \sum_{i=1}^{N} \omega_i (IV_i^P(\Omega) - IV_i^M)^2, \quad (10)$$

where $IV_i^P(\Omega)$ denotes the model-implied volatility for the i-th option, and IV_i^M signifies the implied volatility inferred from market option prices via calculation with the MATLAB function *blsimpv*.

In both objective function f_1 and f_2, the choice of weights ω_i plays a crucial role in the calibration process. Following [7], this study adopts an ask-bid spread-based weighting scheme, i.e., $\omega_i = \frac{1}{|ask_i - bid_i|}$, where the ask-bid spread $ask_i - bid_i$ reflects the discrepancy between buyers and sellers. This weighting scheme assigns greater importance to more liquid option contracts with narrower bid-ask spreads.

3 Methodology

3.1 Multi-objective Genetic Algorithm

The multi-objective genetic algorithm constitutes a class of population-based stochastic search methods that have been widely employed in evolutionary computation. Under the inspiration of the principles of natural selection, the multi-objective genetic algorithm explores the solution space through the iterative application of genetic operators such as selection, crossover, mutation, and elitism. The overall procedure of the multi-objective genetic algorithm implemented in this study is illustrated in Fig. 1.

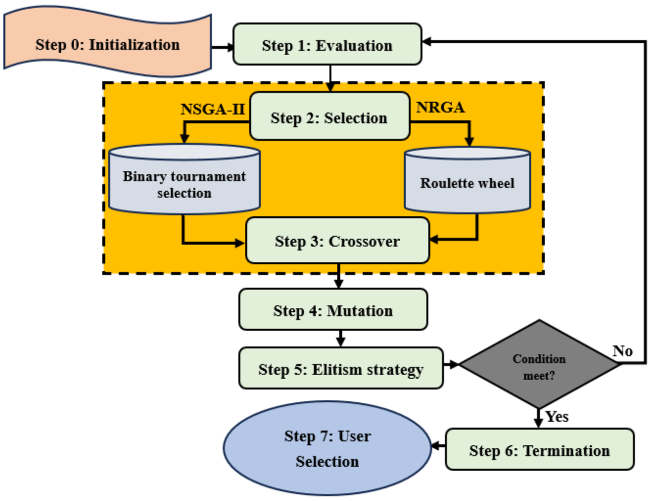


Fig. 1. The flowchart of multi-objective genetic algorithm via NSGA-II and NRGA.

As illustrated, the multi-objective genetic algorithm consists of initialization, evaluation, selection, crossover, mutation, elitism strategy, and termination. Unlike a conventional single-objective genetic algorithm, which optimizes only one objective function, a multi-objective genetic algorithm seeks a set of trade-off solutions that represent different compromises among competing objectives. To accommodate multiple objectives, the multi-objective framework incorporates Pareto-based ranking mechanisms and diversity preservation techniques, as illustrated by the yellow dashed box in Fig. 1. In this extension, the evolutionary process, particularly the selection stage (Step 2), is adapted to evaluate and propagate solutions based on their relative performance across multiple objectives, rather than a single scalar fitness measure. Further details regarding the implementation of each step are summarized below.

Step 0 (Initialization): An initial set of candidate solutions is generated randomly within the predefined parameter bounds. Each chromosome encodes a complete parameter vector $\Omega = (\kappa, \theta, \sigma, \rho, V_0)$, where each gene corresponds to one parameter of the Heston option pricing model. Gene values are real-valued and constrained within admissible ranges, so that each chromosome represents a feasible parameter configuration.

Step 1 (Evaluation): Each candidate solution is evaluated according to performance measures. The resulting objective values are used to assess solution quality and to construct a provisional set of non-dominated solutions.

Step 2 (Selection): Each solution is evaluated using the objective functions, and individuals are ranked based on Pareto dominance and associated ranking mechanisms in NSGA-II and NRGA. Based on this ranking, individuals are selected to form a mating pool, with preference given to higher-quality solutions.

Step 3 (Crossover): Crossover is applied to pairs of parent solutions with probability $p_c = 0.9$, whereby the resulting offspring inherit characteristics from both parents. This mechanism improves global exploration of the search space.

Step 4 (Mutation): Each gene in the offspring population is subject to mutation with probability $p_m = 0.1$ [15]. By introducing random perturbations, the mutation operator increases local search capability and helps maintain population diversity.

Step 5 (Elitism strategy): To construct the next generation, parent and offspring populations are merged, and the best individuals are selected according to non-dominated sorting and diversity criteria until the population size is restored. This elitist replacement strategy ensures the preservation of high-quality solutions.

Step 6 (Iteration Termination): The evolutionary process continues until a stopping criterion is met. In this study, the algorithm terminates upon reaching a maximum number of generations, denoted by $MAXGEN = 200$.

Step 7 (User Selection): Upon termination, the non-dominated solutions on the first Pareto front are returned, representing different trade-offs among the competing objectives and serving as candidate parameter estimates for subsequent analysis.

3.2 Evolutionary Algorithms

3.2.1 NSGA-II

NSGA-II [27] is one of the most widely used and effective methods. The name NSGA-II refers to the incorporation of two key operators into the genetic algorithm framework for solution ranking, namely fast non-dominated sorting and crowding distance. Generally, NSGA-II uses binary tournament selection technique for selecting each parent, and then two random solutions are picked up. An individual with a lower front rank will be chosen. In the case that both solutions come from the same front, the one with a higher crowding distance is the winner.

3.2.2 NRGA

NRGA [28] is a population-based multi-objective genetic algorithm capable of addressing nonconvex, nonlinear, and discrete optimization problems. It integrates a Pareto-based population ranking scheme with a ranked-based roulette wheel selection mechanism as its core. In NRGA, each individual is first assigned to a non-dominated front, and its relative position within the front is determined according to the crowding distance measure. Selection proceeds by first identifying the appropriate non-dominated front and then choosing individuals within that front based on their ranking information. This mechanism ensures that both convergence toward the Pareto front and diversity among solutions are maintained during the evolutionary process.

Compared with NSGA-II, NRGA differs primarily in its selection strategy and the procedure used to rank and select individuals for the next generation. Specifically, NRGA replaces the crowded tournament selection operator adopted in NSGA-II with a ranked-based roulette wheel selection (RRWS) scheme [33]. Under this strategy, individuals with superior fitness receive higher selection probabilities, thereby increasing their likelihood of contributing to the next generation.

3.3 Algorithm Performance Metrics

To quantitatively compare the performance of the NSGA-II and NRGA algorithms in solving the proposed multi-objective optimization problem, four algorithm-level performance metrics are considered.

(1) Number of Non-Dominated Solutions

The number of non-dominated solutions (NNS) refers to the cardinality of the Pareto-optimal front. A larger number of non-dominated solutions indicate higher solution diversity and stronger exploration capability of the algorithm.

(2) Mean Ideal Distance

The mean ideal distance (MID) measures the average distance between the Pareto-optimal solutions and the ideal point. A smaller MID value implies better convergence performance, as the solutions obtained are, on average, closer to the ideal point [34]. The MID is defined as $\mathrm{MID} = \frac{1}{n}\sum_{i=1}^{n} D_i$, where D_i denotes the distance between solution i and the ideal point of solutions, and n is the number of solutions in the Pareto-optimal front. Lower MID values indicate superior solution quality.

(3) Spacing Metric

The spacing metric (SM) evaluates the uniformity of non-dominated points in solution space. This index is calculated based on the formulation proposed by [35] and is given by $\mathrm{SM} = \frac{\sum_{i=1}^{n-1}|\bar{d}-d_i|}{(n-1)\bar{d}}$, where d_i denotes the distance between two consecutive solutions on the Pareto-optimal front, $\bar{d}$ is the average of all d_i, and n is the number of solutions. A smaller SM value indicates a more evenly distributed Pareto front.

(4) Diversity Metric

The diversity metric (DM), originally proposed by [36], measures the overall spread of solutions along the Pareto front. A higher DM value indicates better dispersion and broader coverage of the objective space. The index is defined as $\mathrm{DM} = \sqrt{\sum_{i=1}^{N} d_i'}$, where $d_i' = \max_j \left\{\sum_{m=1}^{M}(f_m^i - f_m^j)^2\right\}$, f_m^i represents the value of the m-th objective function for solution i, M is the number of objectives, and N is the number of Pareto-optimal solutions.

4 Experimental Analysis

This section applies the multi-objective genetic algorithm to calibrate the Heston model in both SSE 50ETF and S&P 500 option markets, and then evaluates its pricing performance across developing and developed market environments. Additionally, the model's robustness is further verified through simulation experiment.

The option data are preprocessed as follows to ensure the reliability of the empirical analysis. First, option contracts with zero trading volume are excluded. Second, call options that violate the no-arbitrage condition are removed. Specifically, options that do not satisfy $C_t \geq \max\left(S_t - Ke^{-r(T-t)}, 0\right)$ are removed from the sample.

To evaluate the effectiveness of the proposed framework, its pricing performance is compared with several benchmark methods. These benchmarks include Nonlinear Least Squares (NLS), Simulated Annealing (SA) [7], conventional GA [15], random sequence Monte Carlo (MC), dual-variable Monte Carlo (DVMC), and control variable Monte Carlo (CVMC). For the Monte Carlo-based approaches, simulations are conducted using 1,000 and 10,000 iterations, respectively. All benchmark methods are implemented under a single-objective optimization setting. That is, option price accuracy is used as the objective function when evaluating pricing performance, while implied volatility accuracy is used when assessing volatility fitting performance.

The performance of each scheme is assessed using the root mean squared error (RMSE) and mean absolute error (MAE), which are defined as

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\hat{y}_i - y_i)^2},\ MAE = \frac{1}{N}\sum_{i=1}^{N}|\hat{y}_i - y_i|,$$

where $\hat{y}_i$ and y_i represent the model-predicted and market-observed option prices, respectively, and N is the number of observations.

4.1 Experimental Results

4.1.1 SSE 50ETF Options

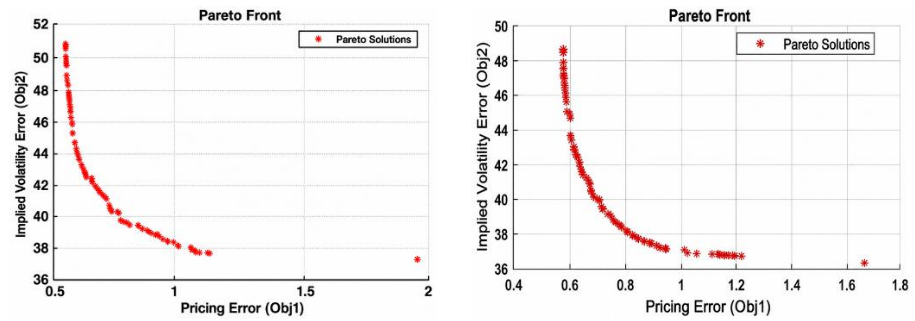
We first investigate the performance of the proposed calibration framework in the SSE 50ETF option market. Option data observed on May 12, 2025, are used for parameter calibration, and the estimated parameters are subsequently applied to price options observed on July 7, 2025. The sample consists of contracts with expiration dates of May 28, June 25, and September 24, 2025, and strike prices (in yuan) ranging from 2.35 to 3.23. On May 12, 2025, the SSE 50ETF closed at 2.76 yuan, and the three-month SHIBOR of 0.01672 is adopted as the risk-free interest rate [13]. All data are sourced from the Bloomberg database.

The NSGA-II and NRGA algorithms are implemented under identical experimental settings, and the corresponding results are reported in Table 1. The results indicate that the two algorithms exhibit broadly comparable performance in the calibration of the Heston model for the SSE 50ETF option market. NRGA achieves negligibly lower values of MID and SM, which implies marginally stronger convergence accuracy, whereas NSGA-II produces insignificantly higher values of NNS and DM, which indicates a larger set of non-dominated solutions and relatively better diversity. Given the comparable performance of the two methods, the subsequent analysis is based on calibration results from both approaches.

Table 1 Algorithm performance from both NSGA-II and NRGA for SSE 50ETF.

Algorithm	NNS	MID	SM	DM
NSGA-II	97	40.714866	0.171019	13.231551
NRGA	95	40.260687	0.105688	12.475561

Fig. 2 presents the Pareto front obtained using both NSGA-II and NRGA, illustrating the inherent trade-off between pricing error and implied volatility error. The monotonic shape of the Pareto front reflects a stable conflict between the two objectives, such that improvements in one dimension are accompanied by deteriorations in the other. TOPSIS [37] with equal weights [38] is then applied to select the best compromise parameter vector, and the resulting estimates are reported in Table 2.



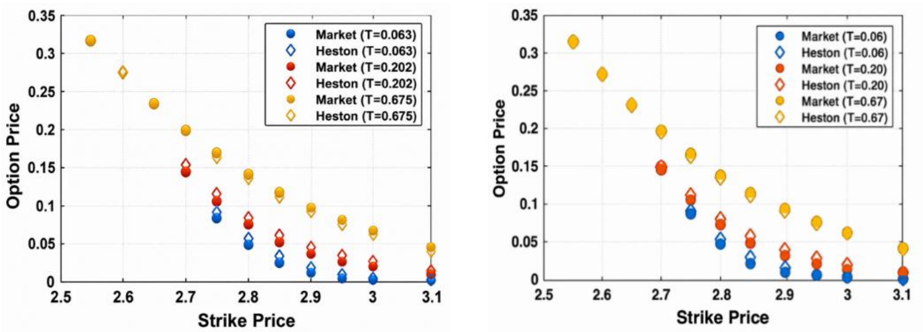
(a) NSGA-II (b) NRGA

Fig. 2. Pareto fronts for the SSE 50ETF. Panels (a) and (b) show the Pareto front obtained by NSGA-II and NRGA, respectively.

Table 2 Parameters estimates for SSE 50ETF based on NSGA-II and NRGA.

	κ	θ	σ	ρ	V_0
NSGA-II	6.87668	0.01345	0.42978	0.59962	0.01614
NRGA	8.23312	0.01349	0.47120	0.60618	0.01624

Using the calibrated parameters, option prices on the SSE 50ETF are evaluated for the out-of-sample date of July 7, 2025. On that date, the index level is 2.83 yuan, and the three-month SHIBOR is 0.01579. The corresponding times to maturity are $T = 0.063$, 0.202, and 0.675. Fig. 3 shows the model-predicted prices with the observed market closing prices across different strike levels for the three maturities.



(a) NSGA-II (b) NRGA

Fig. 3. Option pricing results for SSE 50 ETF options on July 7, 2025. Note: Circles denote the true option price, and rhombuses denote the predicted values of the option price obtained by the calibrated parameters. The x-axis is the strike price, and the y-axis denotes the option price. Panel (a) and (b) show results based on NSGA-II and NRGA, respectively.

The pricing results demonstrate a high degree of consistency between the model-implied prices and the observed market prices across all strike levels. This consistency holds for short-, medium-, and long-maturity options, indicating that the calibrated model effectively captures the term structure of option prices. To further assess the performance of the proposed framework, these results are compared against those from several single-objective benchmark methods, as summarized in Table 3.

Table 3 Comparison results of RMSE and MAE for SSE 50ETF.

Methods		RMSE		MAE	
Monte Carlo	Simulations	Price	IV	Price	IV
MC	1000	0.00649	0.01710	0.00567	0.01463
	10000	0.00560	0.01659	0.00487	0.01384
DVMC	1000	0.00519	0.01642	0.00454	0.01306
	10000	0.00481	0.01585	0.00412	0.01252
CVMC	1000	0.00526	0.01602	0.00434	0.01252
	10000	0.00475	0.01584	0.00413	0.01259
NLS		0.07554	0.04036	0.06532	0.03545
SA		0.00563	0.02886	0.00480	0.02153
GA		0.00517	0.02888	0.00439	0.02208

NSGA-II	**0.00463**	0.01594	**0.00383**	0.01222
NRGA	0.00471	**0.01563**	0.00406	**0.01221**

Note: The minimum values are in bold.

The findings show that the proposed multi-objective framework achieves improved pricing accuracy relative to single-objective approaches. For example, RMSE values for option prices and implied volatility are generally lower under the multi-objective framework than those obtained from the benchmark single-objective methods. Given that NRGA and NSGA-II exhibit comparable performance in parameter calibration, their pricing results are also closely aligned, further indicating that the observed improvements are attributable to the multi-objective formulation rather than the specific choice of evolutionary algorithm. These results highlight the limitations of single-objective calibration, where improvements in one criterion often come at the expense of others. By contrast, the proposed multi-objective framework achieves a more balanced fit, reducing implied volatility errors while maintaining pricing accuracy.

4.1.2 S&P 500 Options

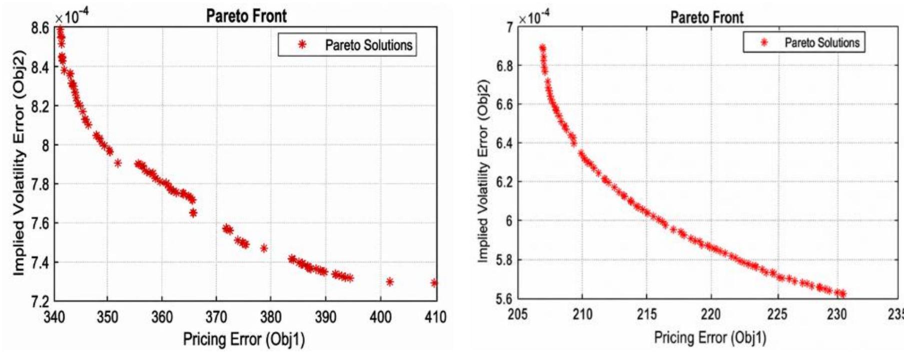
We next assess the performance of the proposed calibration framework in the S&P 500 option market. Options observed on March 12, 2024 are used for in-sample parameter calibration, while contracts recorded on May 7, 2024 serve as the out-of-sample pricing evaluation set. The dataset includes options with maturities on March 22, May 17, and June 21, 2024, and strike prices ranging from 5110 to 5480 index points in increments of 5. On March 12, 2024, the S&P 500 index closed at 5175.27, with the U.S. Treasury constant maturity yield standing at 0.0548. All data are sourced from the Bloomberg database, and contracts with zero trading volume are excluded.

Both NSGA-II and NRGA are applied to the S&P 500 index. As reported in Table 4, the two algorithms produce comparable outcomes, indicating that the results are robust to the choice of optimization method.

Table 4 Algorithm performance from both NSGA-II and NRGA for S&P 500.

Algorithm	NNS	MID	SM	DM
NSGA-II	100	362.797766	1.342717	68.511819
NRGA	100	216.543478	0.163235	23.387757

Fig. 4 presents the Pareto front obtained from both methods, revealing a trade-off between pricing errors and implied volatility errors consistent with that observed in Fig. 2. The TOPSIS-based compromise parameter estimates selected for pricing analysis are reported in Table 5.



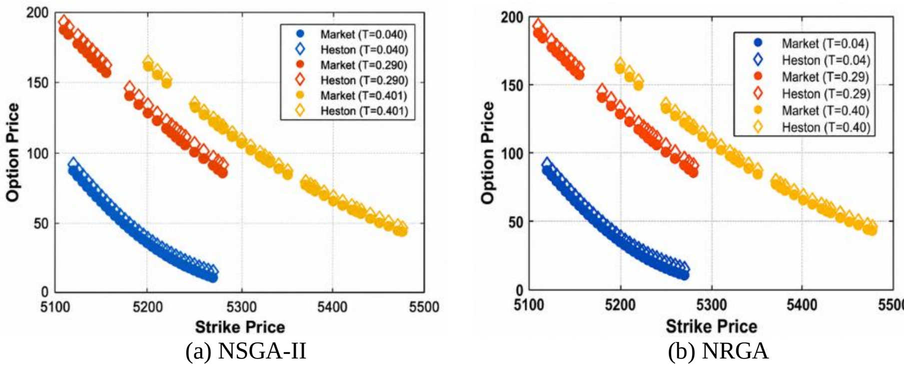
(a) NSGA-II (b) NRGA

Fig. 4. Pareto fronts for the S&P 500. Panels (a) and (b) show the Pareto front obtained by NSGA-II and NRGA, respectively.

Table 5 Parameters estimates for S&P 500 based on NSGA-II and NRGA.

	κ	θ	σ	ρ	V_0
NSGA-II	4.20616	0.00352	0.05707	0.12819	0.01032
NRGA	7.56641	0.00513	0.01425	0.46746	0.01053

The calibrated parameters are subsequently used to price European call options on May 7, 2024, when the S&P 500 index stood at 5187.70 and the risk-free rate was 0.0545, covering maturities of T = 0.040, 0.290, and 0.401. As illustrated in Fig. 5, the model-implied option prices exhibit a high degree of consistency with observed market prices. Pricing errors remain small even at relatively high index levels and across a wide strike range, attesting to the strong pricing accuracy of the calibrated Heston model.



(a) NSGA-II (b) NRGA

Fig. 5. Option pricing results for S&P 500 on May 7, 2024. Note: Circles denote the true option price, and rhombuses denote the predicted values of the option price obtained by the calibrated parameters. The x-axis is the strike price, and the y-axis denotes the option price. Panel (a) and (b) show results based on NSGA-II and NRGA, respectively.

Table 6 reports the RMSE and MAE for S&P 500 options across the competing calibration methods. The proposed multi-objective framework systematically outperforms conventional single-objective approaches. Shifting from the standard GA to NRGA reduces price RMSE by 10.9% and volatility RMSE by 14.9%, while NSGA-II achieves reductions of 7.9% and 14.7%, respectively, consistent with the gains observed under NRGA. Collectively, these results confirm that the multi-objective approach enhances predictive accuracy along both dimensions, further corroborating the robustness of the empirical evidence documented in the SSE 50ETF market.

Table 6 Comparison results of RMSE and MAE for S&P 500.

Methods		RMSE		MAE	
Monte Carlo	Simulations	Price	IV	Price	IV
MC	1000	5.69034	0.00997	4.77339	0.00774
	10000	4.47592	0.00886	4.16480	0.00711
DVMC	1000	4.95872	0.00871	4.60323	0.00729
	10000	4.29819	0.00859	4.12606	0.00701
CVMC	1000	4.25742	0.00856	4.07861	0.00697
	10000	4.18545	0.00854	4.02426	0.00692
NLS		3.84202	0.17159	3.81199	0.13995
SA		4.05623	0.00761	3.86629	0.00618
GA		4.06668	0.00763	3.89715	0.00602
NSGA-II		3.76853	0.00665	3.54649	**0.00563**
NRGA		**3.66557**	**0.00664**	**3.54548**	0.00564

Note: The minimum values are in bold.

4.3 Simulation Study

In this section, a Monte Carlo simulation study is conducted to further assess the robustness of the proposed calibration framework. The initial underlying asset price is set to $S_0 = 2.8$, and the risk-free interest rate is fixed at $r = 0.025$. Option maturities T correspond to 20, 40, 60, 80, and 100 calendar days. For each maturity, strike prices K are obtained by inverting target delta values of 0.10, 0.25, 0.50, 0.75, and 0.90 under the Black-Scholes model. The true Heston parameters are set to $(\kappa, \theta, \sigma, \rho, V_0) = (6.0, 0.02, 0.45, 0.6, 0.02)$. Given these parameters, the true option price is generated by $P^{true} = (S_0 P_1 - Ke^{-r\mathrm{T}} P_2)(1 + 0.01\varepsilon)$, where P_1 and P_2 are obtained according to Equation (6), and ε represents independently and identically distributed random errors generated from three distributions: the standard normal distribution $N(0,1)$, the Student's t-distribution with five degrees of freedom $t(5)$, and the uniform distribution on $U(-1,1)$. The number of simulation replications is 500. The RMSE is used as the comparison criterion for parameter calibration, while the ARMSE and AMAE, defined as the average RMSE and average MAE across 500 simulation replications, are adopted as the comparison criteria for option pricing performance.

TOPSIS with equal weights assigned to both calibration objectives is employed within the multi-objective framework. The simulation study is conducted to evaluate and compare the calibration performance of NSGA-II and NRGA within the proposed bi-objective framework. For comparison, NLS, SA, and conventional GA are included as single-objective benchmarks. The simulation results are reported in Tables 7 and 8.

Table 7 Results of RMSE for parameter calibration.

	Method		κ	θ	σ	ρ	V_0
$N(0,1)$	NSGA-II		**7.2870**	0.1763	0.3771	0.6303	0.0069
	NRGA		7.5696	0.2111	0.3796	0.7046	0.0083
	GA	Price	8.2506	0.0669	0.4956	0.8003	0.0123
		IV	7.6241	0.0999	0.4432	**0.6247**	0.0113
	SA	Price	8.1318	0.2374	0.3062	0.8187	0.0059
		IV	7.6948	0.2869	**0.2976**	0.7895	**0.0041**
	NLS	Price	13.5698	**0.0397**	0.9791	1.4975	0.0200
		IV	13.6176	0.0406	0.8641	1.2278	0.0455
$t(5)$	NSGA-II		**7.2166**	0.1652	0.3862	0.6756	0.0072
	NRGA		7.5006	0.1932	0.3735	**0.6754**	0.0078
	GA	Price	8.4189	0.0604	0.5178	0.8143	0.0126
		IV	7.7691	0.0766	0.4632	0.6780	0.0119
	SA	Price	7.8122	0.2454	0.2995	0.7769	0.0075
		IV	7.9885	0.2764	**0.2953**	0.7768	**0.0041**
	NLS	Price	13.5830	**0.0394**	0.9755	1.4974	0.0194
		IV	13.6879	0.0402	0.8622	1.2200	0.0455
$U(-1,1)$	NSGA-II		**7.5328**	0.1867	0.3813	0.6904	0.0071
	NRGA		7.5415	0.1996	0.4069	0.6784	0.0080
	GA	Price	7.4984	0.0836	0.4758	0.6655	0.0118
		IV	8.0592	0.0712	0.7178	**0.2539**	0.0111
	SA	Price	7.6962	0.2476	0.2955	0.8009	0.0057
		IV	8.0213	0.2583	**0.2891**	0.8272	**0.0037**
	NLS	Price	13.5726	**0.0395**	0.9766	1.5092	0.0188
		IV	13.7009	0.0424	0.8818	1.2500	0.0479

Note: The minimum values are in bold.

Table 8 Results of ARMSE and AMAE for option pricing.

	Method	ARMSE		AMAE	
		Price	IV	Price	IV
$N(0,1)$	NSGA-II	**0.00647**	**0.01980**	**0.00318**	**0.01020**
	NRGA	0.00835	0.02344	0.00430	0.01311
	GA	0.01186	0.03225	0.00695	0.01916
	SA	0.00699	0.02096	0.00524	0.01602
	NLS	0.01820	0.10556	0.01478	0.08500
$t(5)$	NSGA-II	0.00709	0.02088	**0.00344**	**0.01080**
	NRGA	0.00819	0.02241	0.00395	0.01189
	GA	0.01212	0.03397	0.00704	0.02057
	SA	**0.00654**	**0.02034**	0.00505	0.01558
	NLS	0.01756	0.10428	0.01444	0.08369
$U(-1,1)$	NSGA-II	0.00706	0.02038	**0.00332**	**0.01027**
	NRGA	0.00810	0.02295	0.00402	0.01216
	GA	0.01281	0.03363	0.00734	0.02029

	SA	**0.00633**	**0.01993**	0.00506	0.01554
	NLS	0.01660	0.10998	0.01390	0.09054

Note: The minimum values are in bold.

In terms of parameter calibration, both NSGA-II and NRGA produce parameter estimates closer to the true values across all five Heston parameters than the single-objective benchmark methods. Single-objective approaches that focus exclusively on either pricing accuracy or implied volatility fitting tend to improve certain parameters at the expense of others, whereas the proposed bi-objective framework yields more balanced parameter estimates without substantially distorting individual parameters.

With respect to option pricing, NSGA-II achieves the lowest AMAE among the competing methods, indicating strong overall pricing accuracy. Both NSGA-II and NRGA exhibit comparable pricing performance, with only minor differences between them. Under the heavy-tailed (Student's t) and uniform distribution scenarios, the ARMSE of NSGA-II are marginally higher than that of SA; however, the difference is negligible and does not constitute an economically meaningful distinction. Overall, both NSGA-II and NRGA demonstrate robust and stable performance across all error distribution specifications.

In summary, the proposed multi-objective framework performs consistently well regardless of whether NSGA-II or NRGA is employed, and the consistency of findings across empirical and simulated data provides convincing evidence of its robustness and stability in both parameter calibration and option pricing.

5 Conclusion

This study addresses the parameter calibration problem of the Heston model through a bi-objective framework integrating NSGA-II and NRGA. By jointly minimizing option pricing errors and implied volatility errors, the proposed approach generates a Pareto front that explicitly characterizes the trade-off between the two competing calibration criteria. The framework is evaluated through empirical analyses on SSE 50ETF and S&P 500 option markets and Monte Carlo simulation experiments. The results show that NSGA-II and NRGA produce broadly consistent parameter estimates, achieving greater pricing accuracy relative to conventional single-objective benchmarks. Overall, the findings demonstrate that the bi-objective formulation offers a more effective and robust approach to Heston model calibration.

Several directions remain for future research. First, extending the proposed framework to American options and other path-dependent derivatives would broaden its practical applicability. Second, relaxing the restrictive assumptions of the standard Heston model, for instance, by incorporating jump diffusion processes or stochastic interest rates, may improve its capacity to capture more complex market dynamics. Third, while this study adopts a bi-objective formulation, future work could extend the framework to a multi-objective setting by incorporating additional calibration criteria such as bid-ask spread penalties or term structure fitting accuracy.

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